Institute for Information Transmission Problems

LABORATORY 4

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DIRECTIONS OF ACTIVITY:

- the Gibbs random fields and Markov chains with local interactions;
- mean-field models of queuing systems;
- fluid models of queuing networks;
- large deviations and its applications;
- queuing systems;
- systems of information transmission, information channels and coding theory;
- algebraic geometry and number theory;
- combinatorial and probabilistic aspects of representation theory;
- modal logics.

MAIN RESULTS

An one-boson space for the polaron system is constructed. This space describes scattering states of one boson on a polaron.

We considered the ferromagnetic Ising model with Glauber spin flip dynamics in one dimension. The external field vanishes, and the couplings are independent identically distributed bounded or unbounded random variables. The long-time behavior of the time auto-correlation function averaged over the disorder was under consideration.

Using the spectral analysis of the transfer matrix for classical lattice spin systems under high temperatures we found invariant subspaces for quasi-particles of various species. The corresponding branches of the transfer matrix spectrum ("the energy of the quasi-particles") have different orders with respect to a small parameter (inverse temperature).

The central limit theorem in the region of small stochasticity for directed polimer in a random environment.

A number of results from one dimensional dynamics based on spectral properties of the Ruelle-Perron-Frobenius transfer operator has been extended to Anosov diffeomorphisms and general expanding or contracting on average random maps on compact manifolds.

Analysis of statistical properties of a family of maps acting in the space of integer valued sequences, which model dynamics of simple deterministic traffic flows has been done. Asymptotic (as time goes to infinity) properties of trajectories of those maps corresponding to arbitrary initial configurations in terms of statistics of densities of various patterns (in the space of sequences) are obtained and weak attractors of these systems and the rate of convergence to them are described.

A class of ferromagnetic bynary models with manyparticle interactions is studied. The classification of these models from the point of view of the drop form of one phase in another one.

Condition on Hamiltonian under which there exists a unique state of non-ideal gas are given.

An asymptotic behavior of an epsilon-entropy of a ellipsoid in a Hamming space (as his dimension increases) is invesigated. Sufficient conditions for the asymptotic exactness of the generalized Hamming bound are obtained. An exact solution of an optimization problem is found. This problem is related to the investigation of asymptotic of the epsilon-entropy of an ellipsoid.

Harmonic functions on certain fractals have benn considered. In particular, for the case of the Sierpinski carpet, an explicit expression involving fractional derivatives has been obtained.

A detailed study of the Gromov-Witten theory for curves has been undertaken. This study is connected with the classical Hurwitz theory of enumeration of maps between curves by their ramification data, which, in turn, is closely connected with symmetric group characters. The results include a proof of the known conjecture concerning a link between the Gromov-Witten theory for the projective line and the Toda hierarchy of nonlinear equations.

The dimer model on periodic bipartite planar graphs has been studied. It is shown that the scaled limit behavior of the model can be described in terms of the so-called amoeba of a suitable plane real algebraic curve, which turns out to be a Harnack curve. This is a generalization of the previous results on the limit behavior of random 3-dimensional partitions.

It has been shown that the distribution function of the first particle in a discrete polynomial ensemble can be obtained through a certain recurrence procedure provided that the difference (or the q-difference) log-derivative of the weight function is rational. In a number of cases the recurrence procedure is equivalent to certain difference or q-difference analogs of Painleve equations. Our approach is based on the formalism of discrete analogs of integrable operators and Riemann-Hilbert problems as developed in our previous works.

An analog of isomonodromy deformations for matrix difference equations has been introduced and studied. It has been shown that both the classical Schlesinger equations and Schlesinger transformations can be obtained from our construction through appropriate limit procedures.

A new relation between Ulam's problem on longest increasing subsequences in random permutations and random matrix ensembles has been studied. New proofs of the Baik-Deift-Johansson theorem and its generalizations due to Baik-Rains has been obtained.

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A family of probability measures on partitions of naturals has been studied. These measures originated in representation theory, and we demonstrate their connections with random matrix theory and multivariate hypergeometric functions. Our measures depend on three parameters including an analog of the beta parameter in random matrix models. Under an appropriate limit transition our measures converge to certain limit measures, which are of the same nature as one-dimensional log-gas with arbitrary value of the parameter beta. It is proved that averages of products of "characteristic polynomials" with respect to the limit measures are given by the multivariate hypergeometric functions of type (2,0).

Codes in real and complex Grassmannians were studied. For such codes the Varshamov-Gilbert and Hamming type bounds on the size were obtained.

The theory of infinite number fields and function fields was constructed. In particular, an analogue of a zeta-function was introduced. The Odlyzko-Serre bounds on discriminants and the Brauer-Siegel theorem on the product of the class number and the regulator are generalized.

Realizations of the category of mixed motives were studied. Analogues of Hilbert theorem proven. Various faithfullness results for realization functors obtained. Conjectures introduced which together with Beilinson conjuctures would imply that the realization functor constructed is actually an equivalence of categories.

Deformations of Picard-Fuchs differential equations were studied. A family of the 3-rd order ordinary differential equations related to 3-dimensional Fano varieties was introduced. The mirror-symmetry predicts the geometricity of this family. For the case of a complete intersection the stronger modularity property is proven.

The connection is established between the growth of multiplicities in branching rules and the complexity of homogeneous spaces. It is shown that the irreducible representation of a reductive group restricted to it's reductive subgroup gives multiplicities growing no faster than the polynomial on the norm of a highest weight, whose degree can be computed as a complexity of an explicitely constructed auxiliary homogeneous space.

The problem of optimal choice of the sequence of isometries of a homogeneous space of a compact Lie group was studied. The estimate on the norm of an averaging operator in the space of functions with zero integral was obtained for arbitrary homogeneous space.

The asymptotic behavior of the eigenvalues and the spectral measures corresponding to a selfadjoint operator assigned to any symmetric sequence of points on a compact Lie group was studied. The convergence of the sequence of spectral measures is proven.

The contribution was made to the Vinberg's theory of θ-groups. The link between that theory and recent results of Springer and Lehrer concerning regular elements of reflection groups was established.

The commuting variety related to an involutory automorphism of a simple Lie algebra was studied. In particular, a sufficient condition for the irreducibility of such variety was obtained. Also, the complete description of the commuting varieties for the symmetric pairs of rank one was given.

Equations satisfied by Cantor derivative operation together with boolean operations were studied for various topological spaces (${\Bbb R^n$; subspaces of ${\Bbb R}$). These equations are described in terms of modal logic. For the corresponging modal logics the finite model property and the decidability was proven.

The problem posed more than 20 years ago by R.Goldblatt is solved: the finite axiom set was constructed and the finite model property was proved for the modal logic of chronological future time in the Minkowski space was studied.

The research on products of modal logics was continued. The finite model property for the products of minimal modal and minimal temporal logic was established. This allows to identify new decidable fragments of classical first-order logic as well as of the equational theory of relational algebras.

The discrete invariants of quadrics were studied. New results on the structure of the Chow groups of the Grassmanians of r-planes on a quadric were obtained. The action of the Steenrod algebra on those groups was described. As an application, the possible dimensions of anisotropic quadratic forms from the given power of the fundamental ideal of even-dimensional forms were computed.

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