LABORATORY 16

Laboratory of Stochastic Dynamical Systems

Head of Laboratory – Dr.Sc. (Mathematics), Prof. **Aleksandr Veretennikov**
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The leading researchers of the laboratory include:

Dr.Sc. (Techn.)	R. Liptser	Dr.	P. Kitsul
Dr.	F. Grigoriev	Dr.	A. Puhalskii
Dr.	O. Gulinsky	Dr.	A. Serebrovskii
Dr.	V. Kistlerov	Dr.	S. Lototsky

Currently R. Liptser, P. Kitsul, A. Puhalskii and S. Lototsky are working abroad remaining the staff members of Laboratory.

ACTIVITY DIRECTIONS AND MAIN RESULTS

Main directions of activity in 2004 was stochastic analysis and its applications:

- large and moderate deviations for stochastic processes;
- large deviations and quasi classical approximations;
- large deviations for invariant measures;
- large and moderate deviations for stochastic graph;
- large and moderate deviations in statistical estimations;
- diffusion approximation for two scaled diffusion;
- diffusion approximation for stochastic differential equations with non smooth coefficients;
- mixing in stochastic equations;
- existence uniqueness and regularity for parabolic stochastic equations with smooth boundary conditions;
- regularity of invariant measure and smoothness of Markov chain generator;
- estimation control, control theory and data processing.

A. Puhalskii studied the asymptotics of large, moderate and normal deviations for the connected components of the sparse random graph by the method of stochastic processes.

The following asymptotics are established: the logarithmic asymptotics of large deviations of the joint distribution of the number of connected components, of the sizes of the giant components, and of the numbers of the excess edges of the giant components. For the supercritical case, the results include the asymptotics of normal deviations and the logarithmic asymptotics of large and moderate deviations of the joint distribution of the number of components, of the size of the largest component, and of the number of the excess edges of the largest component. For the critical case, the logarithmic asymptotics of moderate deviations of the joint distribution of the sizes of connected components and of the numbers of the excess edges are derived. Some related asymptotics are also established. The proofs of the large and moderate deviation asymptotics employ methods of idempotent probability theory. In particular, the results significantly extend and give much deeper insights into (a) a phase transition which had earlier been observed by Stepanov when the edge probability is $2/n$, and (b) a large deviations principle for the size of the giant component which had earlier been obtained by O'Connell. Novel techniques are introduced, including a new "stochastic process" representation for the component structure. These results is of importance both to the random graphs theory and also to statistical physics.

A. Puhalskii also concerned the issue of obtaining the large deviation principle for solutions of stochastic equations with possibly degenerate coefficients. Specifically, he explored the potential of the methodology that consists in establishing exponential tightness and identifying the action functional via a maxingale problem. It was first shown that under certain continuity hypotheses existence and uniqueness of a solution to a maxingale problem of diffusion type are equivalent to Luzin weak existence and uniqueness, respectively, for the assosiated idempotent Ito equation. Consequently, if the idempotent equation has a unique Luzin weak solution, then the action functional is specified uniquely, so the large deviation principle follows.

Two kinds of application are considered. Firstly, we obtain results on the logarithmic asymptotics of moderate deviations for stochastic equations with possibly degenerate coefficients which, as compared with earlier results, relax the growth conditions on the coefficients, permit certain non-Lipshitz-continuous coefficients, and allow the coefficients to depend on the entire past of the process and to be discontinuous functions of time. The other application concerns multiple-server queues with impatient customers.

S. Lototsky proposed new method for constructing a generalized solution for stochastic differential equations. The method is based on the Cameron-Martin version of the Wiener Chaos expansion and provides a unified framework for the study of ordinary and partial differential equations driven by finite- or infinite-dimensional noise with either adapted or anticipating input. Existence, uniqueness, regularity, and probabilistic representation of this Wiener Chaos solution is established for a large class of equations. A number of examples are presented to illustrate the general constructions. A detailed analysis is presented for the various forms of the passive scalar equation and for the first-order Ito stochastic partial differential equation. Applications to nonlinear filtering if diffusion processes and to the stochastic Navier-Stokes equation are also discussed.

S. Lototsky studied time evolution of a passive scalar in a turbulent homogeneous incompressible Gaussian flow. The turbulent nature of the flow results in non-smooth coefficients in the corresponding evolution equation. A strong, in the probabilistic sense, solution of the equation is constructed using Wiener Chaos expansion, and the properties of the solution are studied. Among the results obtained are a certain regularity of the solution and Feynman-Kac-type, or Lagrangian, representation formula. The results apply to both viscous and conservative flows.

A. Veretennikov studied approximations and mixing for stochastic differential equations, invariant measures for Markov processes, and McKean-Vlasov equations.

New more precise upper bounds on sub-exponential mixing rates are obtained. Lower mixing bounds is proved. The last result shows optimal properties of upper bounds obtained earlier. New relaxed conditions for regularity of invariant diffusion densities with respect to a parameter were obtained.

The existence of invariant measures for McKean-Vlasov stochastic equations has been proved under certain assumptions.

R. Liptser studied asymptotic properties and stability of nonlinear filters for ergodic and nonergodic signals. New results are obtained on existing of limiting distribution for time-nonhomogeneous countable Markov process. Asymptotic stability of nonlinear filters in non-mixing case and Wonham filter for nonergodic signals have been investigated.

O. Gulinsky studied connection between the theory of ultracontractive semigroup and the theory of u-bounded positive operators. Originally the concept of hypercontractive and ultracontractive semigroup was introduced for the case of harmonic oscillator in quantum field theory and Scrodinger operators. And the same time these ideas are useful in general theory of Markov semigroup and are closely connected to the mixing properties of Markov processes. Theory of positive operators deals with the problem of existing of positive eigen functions. For the spaces with good cone u-bounded operators have positive eigen function corresponding to positive eigen value which in case is equal to spectral radii.

It is proved that under certain conditions ultracontractive properties of semigroup e^{-tH} are equivalent to u-boundness of operators in semigroup for each t>0.

Teaching. Moscow Institute of Physics and Technology: O. Gulinsky, A. Serebrovskii, and F. Grigoriev; Universities abroad: A. Veretennikov, R. Liptser, A. Puhalskii, P. Kitsul, S. Lototski.

International collaboration. Fruitful collaboration is established with the probability group of the LATP CMI Universite de Provence, Marseille, France, and, in particular, with Professor Etienne Pardoux as its leader.

We also have close contacts with Universite Paris 6 (Professors Jean Jacod and Pierre Priouret); Universite du Main in France (Professor Yuri Kutoyants); Weierstrass Institute for Applied Analysis and Stochastics – WIAS, Berlin, Germany; the Univerity of Warwick, UK (Professor David Elworthy); Mathematical Institute of the University of Copenhagen; University of Trier (Professor Dieter Baum); University of Wuerzburg (Professor Elart von Collani), and some others.

Conference talks and seminar talks. A. Veretennikov, conference talks: "New techniques in applied stochastics" (Finland, 2004) and "MCMC2" Conference (France 2004) on McKean-Vlasov stochastic equations; seminar talks: Juan-les-Pins, Le Mans, Rennes (France), Leeds, Oxford (UK), Helsinki (Finland), Moscow/IITP (Russia). S. Lototsky, conference talks: "Incompressible Turbulent Transport: a Wiener Chaos Approach", 4th Southern California Applied Mathematics Symposium, Claremont, April 2004; Turbulent Transport via SPDE and Wiener Chaos, International Workshop on Nonlinear Dynamics and Stochastic Partial Differential Equations, Beijing, China, May 31, 2004; seminar talks: Nonlinear Filtering in Correlated Noise: "A Wiener Chaos Approach",Scientific Systems Company, Inc., Woburn, MA, April 15, 2004; "Turbulent Transport and Wiener Chaos", Finance and Stochastics Seminar, Boston University, April 16, 2004; "Stochastic Equations with Space-Only Noise", Probability/Statistics Seminar, University of California, Santa Barbara, October 19, 2004.

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PUDLICATIONS IN 2004

Articles

1. Abramov V., Liptser R. On Existence of limiting distribution for time-nonhomogeneous countable Markov process // Queueing Systems. 2004. V. 46. P. 353-361.
2. Baxendale P., Chigansky P., Liptser R. Asymptotic stability of the Wonham filter: ergodic and nonergodic signals // SIAM Journal on Control and Optimization/ 2004. V. 43. No 2. P. 643-659.
3. Chigansky P., Liptser R. Stability of nonlinear filters in non-mixing case // Annals of Applied Probability. 2004. V. 14. No. 4. P. 2038-2056.
4. Klokov S.A., Veretennikov A.Yu. Subexponential mixing rate for a class of Markov diffusions // J. Math. Sci. September 2004. 123(1). P. 3816-3823.
5. Klokov S.A., Veretennikov A.Yu. Subexponential mixing rate for Markov processes // Math. Comm. 2004. No. 9. P. 9-26.
6. Lototsky S.V., Rozovskii B.L. Passive Scalar Equation in a Turbulent Incompressible Gaussian Velocity Field // Russian Mathematical Surveys. 2004. V. 59. No. 2. P. 297-312.
7. Veretennikov A.Yu. On approximations of diffusions with equilibrium // Helsinki University of Technology, Institute of Mathematics Reports C17 (2004); an electronic version at http://www.math.hut.fi/visitors0405/AVslides.pdf.

In print

1. Lototsky S.V., Rozovskii B.L. Stochastic Differential Equations: A Wiener Chaos Approach. (A review) To appear in the volume in honor of A. N. Shiryaev.
2. Lototsky S.V., Rozovskii B.L. Wiener Chaos Solutions of Linear Stochastic Evolution Equations. Submitted to Annals of Probability on July 2, 2004.
3. Puhalskii A. On some degenerate large deviation problems, to appear in Electronic Journal of Probability.
4. Puhalskii A. Stochastic processes in random graph, to appear in The Annals of Probability.