

LABORATORY 4

Dobrushin Mathematics Laboratory

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DIRECTIONS OF ACTIVITY:

- spectral analysis of generators in stochastic model of mathematical physics;
- Gibbs random fields and Markov chains with local interactions;
- mean-field models of queueing systems;
- large deviations and its applications;
- queueing systems;
- combinatorial and probabilistic problems for information transmission and protection in modern communication systems;
- algebraic geometry and number theory;
- combinatorial and probabilistic aspects of representation theory;
- modal logics.

MAIN RESULTS

For a stochastic model (birth-annihilation model) the one-particle subspace in a configuration space of continuous gas is constructed.

Two leader invariant subspaces for a generator of a Blume-Capel stochastic model (model with nearest neighborhood interaction and three-value spin) are constructed in a high-temperature region. It is shown that on each these subspaces the generator is unitarily equivalent to the operator of production on a bounded function.

For a weakly bounded coupling of a free general quantum lattice system it is proved that if the free system has a non-degenerated bound state and a spectral gap, then the coupled system also has a non-degenerated bound state and a spectral gap.

The central limit theorem is proved for a time-distribution of the end-point of a random walk for arbitrary values of the coupling parameter.

For M/G/1 foreground-background processor-sharing queue with minimal attained service time (a simplified version of TCP/IP protocol) the time-dependent distribution of attained service times is found.

For asymptotic dynamic of infinite-particle system on one-dimensional lattice under the action of a constant force and a friction force it is proved the existence of two different ergodic phases corresponding to two critical values of a particle density and the structure of these phases is studied. The existence of phase transition with an hysteresis is proved and explicit expressions for the life-time of a single particle cluster and the average motion speed are obtained.

Pseudobilliard systems with a special reflection law are introduced and studied. It is shown that such systems may have chaotic, stable and neutral behavior that may coexist in one system.

Iterative stochastic algorithms are proposed for image processing. These algorithms are based on properties of diffusive dynamics (annealing procedure). A number of approximation schemes for numerical solutions of the recovery problem are considered. The convergence of Markov chains to a continuous process is investigated, conditions providing the ergodicity of the Euler approximation scheme are stated.

Queuing systems with two servers and dynamic routing are investigated. Numerical experiments showing a good coincidence of results (obtained with help of large deviations and numerical calculations) are carried out.

A model of magnetostriction is developed. This phenomenon signifies that some substances can discontinuously change their form and size depending on exterior parameters – temperature or magnetic field.

The existence of a first order phase transition is strictly proved for some statistical models with continuous symmetry. The existence of a discontinuous transition is proved for some kind of models with a large entropy.

The problem of 3D-crystal growth is discussed. A hypothesis about a discontinuous growth is stated. The hypothesis is proved for a model “solid-on-solid”.

The validity of the Poisson hypothesis (about the asymptotic behavior of large queuing systems) is established in a naturally general class of time-service distributions. Frameworks of the hypothesis (outside of which it is broken) are indicated,

It is proved that certain difference analogs of the Painleve equations, which have numerous applications in mathematical physics, can be interpreted as isomorphisms of modules of d -connections on the projective line with given singularities. The most general difference Painleve equation known so far is derived; it degenerates to both difference Painleve V and classical (differential) Painleve VI equations.

Odd analogs of the classical and quantum family algebras are introduced and studied. As an example, the structure of a g -module on the exterior power of the adjoint representation for the Lie algebras A_3 , A_4 , G_2 is explicitly determined.

A new method for computing correlations functions of random point processes was found. This method allows one to obtain simpler proofs of two known results, the Eynard-Mehta theorem and the Okounkov-Reshetikhin formula for the correlation kernels of Schur processes. The new method also make it possible to obtain Pfaffian analogs of these results.

Institute for Information Transmission Problems

A family of Markov processes on the set of Young diagrams is constructed. It is proved that the correlation functions of these processes have determinantal structure, and the corresponding correlation kernels are explicitly found. The results show that the recently discovered surprising analogy between asymptotic properties of random partitions arising in representation theory and those of spectra of random matrices extends to the associated time-dependent models. A connection with the classical results of Karlin and McGregor is also established (those results, published in the fifties, concern the birth-death processes associated to various systems of orthogonal polynomials).

Two survey papers, based on talks delivered at the 4th European Congress of Mathematics (Stockholm, 2004), were prepared. Their subjects are: (1) connection between enumerative geometry of algebraic curves and random surfaces, and (2) application of random point processes in representation theory.

Linear codes correcting errors associated with multi-dimensional projective manifolds on a finite field are considered. Codes on Shubert submanifolds in Grassman manifolds (Shubert codes) are investigated. Explicit formulas for the distance and dimension of an arbitrary Shubert code are obtained. The hypothesis about the minimal distance is verified for the case of codes associated with Shubert divisors.

Complex spaces with action of compact Lie group (extending spherical algebraic manifolds) are investigated. For such spaces the simplicity of representation spectrum of a given group in linear fibering cut is proved.

A ring of algebraic cobordisms of Pfister quadric is calculated. It is shown that this ring has no rotating.

The irreducibility of commutator manifolds for some simple Lie algebras involutions (i.e. for some symmetric spaces) is proved. The case for symmetric spaces of rank 1 is completely investigated.

A series of papers is devoted to the investigation of ideals in Borel subalgebra of simple Lie algebras. Obtained results are related to combinatorial aspects of the theory. The relation of ideal with properties of Weyl affine group elements is investigated.

New results about finite-range approximability of productions of modal logics are obtained. A new method of the proof for the finite-range approximability is constructed. A series of open problems about finite-range approximability of many-dimensional modal logics is solved as well as about the theory of binary relations with Boolean operations and compositions with fixed relations and their inverse ones.

The second order asymptotic for the mutual information between inlet and outlet signal in the case when the ratio signal/noise tends to zero is found for the first time and for a very general model of the communication channel.

The asymptotic of an ellipsoid ϵ -entropy in the Euclidean space is found under the assumption that the dimension of the space tends to infinity. The asymptotic depends only on the volume of subellipsoid with axes having the length greater than two ϵ .

"Good" codes (i.e. with asymptotically non-zero rate) with polynomial (on code length) complexity "parents identification" are constructed. A new solution for the problem of "digital imprints" is proposed for the case of two users coalition.

Researchers of the laboratory are teaching in institutes of Moscow (in particular, Moscow State University and Independent Moscow University). Under their leadership 4 post-graduate students are working. One Ph.D. thesis is defended.

A prize of the European mathematical society is awarded to A. Yu. Okunkov for results in asymptotical combinatorial calculus with applications to the topology of module spaces, the ergodic theory, the random surfaces theory and the algebraic geometry.

GRANTS FROM:

- **Grant of the President of Russia for support of leading scientific schools (No. NS-934.2003.1):** school by R. A. Minlos.
- **Russian Foundation of Basic Research (No. 02-01-00444):** "Gibbs States and Dynamic Systems". Project leader R. A. Minlos.
- **Russian Foundation of Basic Research (No. 03-01-00098):** "Combinatorial and Probabilistic Problems of Information Transmission and Protection for Modern Communication Systems". Project leader L. A. Bassalygo.

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